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Propagation of Love waves at the interface of initially stressed transversely isotropic poroelastic layer and in-homogeneous half-space

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Abstract: A detailed study on propagation of love waves at the interface of initially stressed transversely isotropic poroelastic layer and in-homogeneous half-space is presented. The equations of motion in poroelastic layer have been formulated following Biot's theory. Solving the equations of motion the dispersion equation for love waves is derived from which the phase velocity of waves has been studied. In addition, it has been noted that there is much influence of initial stress and homogeneous parameters on phase velocity. Numerical work has been worked out and results are presented graphically. Different effects on phase velocity have been seen for the two homogeneous parameters. The homogeneity of the lower half-space affects phase velocity, with velocity being lower in a homogeneous medium compared to an inhomogeneous one. In a similar vein, both the layer's and the half-space's initial stresses have had a completely different effect on the phase velocity. Phase velocity is strongly affected by initial stress levels in the lower half-space, where higher stress values are correlated with lower velocity.

Key words: Love waves, in-homogeneous, transversely isotropic, phase velocity, lower half-space, wave number, initially stressed.

1. Introduction

Love waves are essential for seismic characterisation and exploration because of their horizontal motion that is perpendicular to the direction of propagation. They are extremely useful for identifying subsurface geological features, fluid distributions, and stress conditions due to their sensitivity to shear characteristics.

Love wave propagation at the interface between an initially stressed transversely isotropic poroelastic layer and an inhomogeneous half-space is a complex phenomenon having broad consequences in various fields. This study provides light on how layered geological formations respond to earthquakes, providing understanding of how seismic energy is distributed and how ground motion is produced. This research helps geotechnical engineers better understand how waves affect the interaction between soil and structure, which helps them build and evaluate infrastructure that is resilient to seismic activity.

Furthermore, this is relevant to the classification of petroleum reservoirs, as the complex interactions between porous geological formations and Love waves have important consequences for the exploration and extraction of hydrocarbons. In order to make well-informed decisions regarding reservoir management strategies, reservoir engineers can obtain important information about reservoir properties like porosity, permeability, and fluid saturation by understanding the subtleties of Love wave propagation in transversely isotropic poroelastic layers.

There are numerous possible biological applications of love wave propagation in porous materials, especially in the fields of biosensors and bio fluidics. The detection of biological substances including proteins, DNA, and cells is possible with love wave-based biosensors. Love wave biosensors are also useful for environmental monitoring, medical diagnostics, and food safety. On tissue engineering and regenerative medicine applications, love wave sensors embedded on porous substrates can be utilised to track cell behaviour. In biological filtration systems, porous materials are often used for the treatment and purification of wastewater. Love wave sensors detect variations in wave propagation caused by the presence of biological agents or contaminants in the fluid, allowing for the monitoring and modification of the performance of these filtration systems.

Loves wave propagation in fluid-filled porous solids was studied by Deresiewicz in (1962). Bhattacharya (1969) presented the propagation of Love waves in a heterogeneous layer bounded between two isotropic elastic layers which are semi-infinite. Shear waves on magnetoelastic monoclinic substrate sandwiched between two isotropic elastic half spaces with rectangular irregularity at the lower interface examined by Chattopadhyay et al. (2009). It is observed that the phase velocity decreases as the irregularity size increases. Additionally, the phase velocity maintains the similar nature as the magnetoelastic monoclinic parameter increases. Nageswaranath et al. (2011) presented the propagation of waves in a viscous liquid layer surrounded by two poroelastic isotropic half-spaces. Three potential bonds between these half-spaces are addressed by taking into account the secular equation with various limiting forms. Kumar(2011) et al. investigated the transmission and reflection of plane waves between two distinct fluid-filled poroelastic half spaces. The variations in ratios of amplitude with angle of incidence are presented graphically. The impact of porosity on the stress components and amplitude ratios has been noted. Manna et al. (2013) discussed propagation of Love wave in heterogeneous elastic half-space and piezoelectric layer. Gupta et al. (2013) proposed a mathematical model to study Love wave propagation in homogeneous and initially stressed heterogeneous half-spaces. Kundu et al (2014) investigated the existence of Love wave propagation in an initially stressed homogeneous layer over a porous half-space with irregular boundary surfaces.

Santimoy Kundu et al. (2014) examined the shear wave propagation in a homogeneous isotropic media encased in a lower heterogeneous medium and an orthotropic medium under initial stress. They noticed that in both homogeneous and inhomogeneous media, the velocity of waves increases with an increase in inhomogeneity parameters, and the velocity of SH-type waves increases with an increase in the initial stress parameter. Love wave propagation in an elastic layer bounded by initially stressed inhomogeneous and orthotropic half-spaces is discussed by Rajneesh Kakar (2015). The impact of inhomogeneity parameter and initial stress parameter on the phase velocity is discussed. It is noted that as the stress parameter increases, the phase velocity increases. The outcomes also show that the wave velocity is considerably influenced by the inhomogeneity. Gupta et al. (2013) presented Love waves propagating at the interface of homogeneous and initially stressed heterogeneous half-spaces.

Love waves in a layered functionally graded piezoelectric structure with an elastic layer in contact with a half-space are studied under initial stress, showing varying phase velocities with wave numbers by Majhi et al. (2016). Vishwakarma et al. (2018) studied the Love wave propagation in an inhomogeneous anisotropic layer superimposed over a half-space under the influence of rigid boundary plane and found that the dispersion equation is a function of phase velocity, wave number, inhomogeneity parameters, and initial stress. Kumhar et al. (2020) examined the dependency of dispersion and damping behavior of Love-type waves on wave number in a heterogeneous dry sandy double layer of finite thickness superimposed on heterogeneous on heterogeneous viscoelastic substrate under the influence of hydrostatic initial stress.

The effect of initial stresses and gravity on the propagation of Love waves have been studied in a porous layer surface over a heterogeneous half-space by Gupta et al. (2021). Porkuian *et al*. (2022) discussed Love wave propagation in a semi-space contact area and a thin layer, analyzing dispersion and phase velocity influenced by layer thickness and material characteristics.

In the present paper, Love waves propagating on transversely isotropic poroelastic layer in contact with in-homogeneous half-space is presented. The dispersion equations of Love waves have been derived and discussed for all the cases under suitable conditions. The phase velocity of plane waves is obtained for a particular model. The effect of initial stresses and inhomogeneity parameters on the propagation of Love waves has been demonstrated through numerical computation of the dispersion equation. The homogeneity of the lower half-space affects phase velocity, with velocity being lower in a homogeneous medium compared to an inhomogeneous one. In a similar vein, both the layer's and the half-space's initial stresses have had a completely different effect on the phase velocity. Phase velocity is strongly affected by initial stress levels in the lower half-space, where higher stress values are correlated with lower velocity. Few particular cases are observed and discussed.

2. Formulation and Solution of the problem:

We examined Love waves on a transversely poroelastic layer with a thickness of *h* that was initially stressed and in contact with an initially stressed in-homogeneous half-space. The origin has been initiated at the interface of layer and half space. Wave motion is towards *x*-axis and the positive *z*-axis is taken towards the interior of the lower half space.

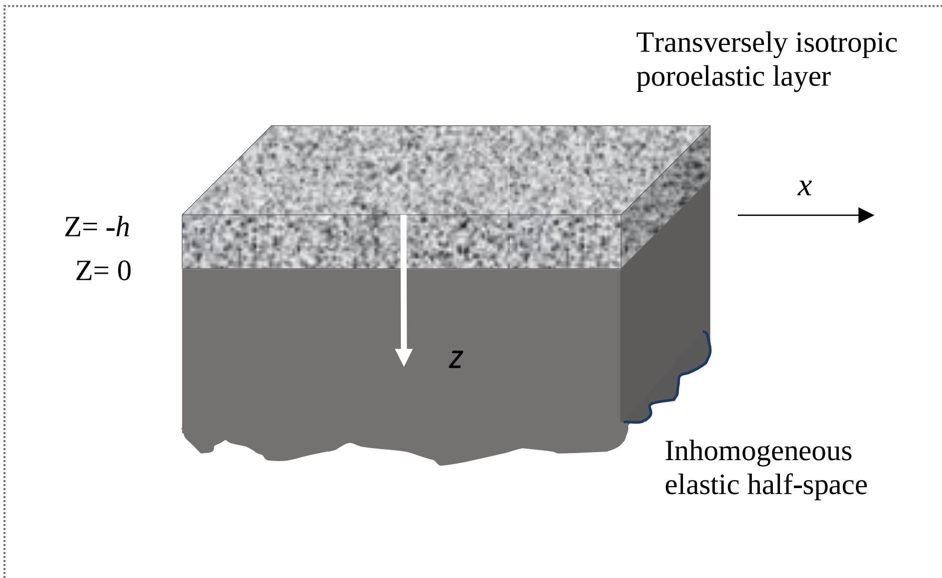

Figure. 1 Transversely isotropic porous layered in-homogeneous half-space

2. 1. Solution in the layer

In the absence of body forces and under initial stress *P*, the dynamic equations of motion (Biot, 1965) in a transversely isotropic poroelastic sheet are

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}+\frac{\partial\sigma_{xz}}{\partial z}-P\left(\frac{\partial\omega_z}{\partial y}-\frac{\partial\omega_y}{\partial z}\right)=\frac{\partial^2}{\partial t^2}(e_{11}u_x+e_{12}U_x)$$

$$\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{yz}}{\partial z}-P\frac{\partial\omega_z}{\partial x}=\frac{\partial^2}{\partial t^2}(e_{11}u_y+e_{12}U_y)$$

$$\frac{\partial\sigma_{zx}}{\partial x}+\frac{\partial\sigma_{zy}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}-P\frac{\partial\omega_y}{\partial x}=\frac{\partial^2}{\partial t^2}(e_{11}u_z+e_{12}U_z)$$

$$\frac{\partial s}{\partial x} = \frac{\partial^2}{\partial t^2}\left(e_{12}u_x + e_{22}U_x\right)$$

$$\frac{\partial s}{\partial y} = \frac{\partial^2}{\partial t^2}\left(e_{12}u_y + e_{22}U_y\right)$$

$$\frac{\partial s}{\partial z} = \frac{\partial^2}{\partial t^2}\left(e_{12}u_z + e_{22}U_z\right) \qquad (1)$$

where, $i, j > 0$ $(i, j = 1,2,3 \ldots .)$ are the incremental stress components; u, v, w are the displacement components in the solid along x, y, z directions respectively, whereas U, V, W are the displacement components in the fluid present in the porous solid. Also, the angular componentsω_x, ω_y , ω_z are given by

$$\omega_x = \frac{1}{2}\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right);\ \omega_y = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right);\ \omega_z = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) \qquad (2)$$

The mass coefficients e_{11} , e_{12}, e_{22} are such that

$e_{11} > 0$, $e_{12} > 0$, $e_{22} > 0$.

Stress, strain relation is a transversely isotropic poroelastic half space under initial P are

$$\sigma_{xx} = (A + 2N + P)e_{xx} + (A + P)e_{yy} + (F + P)e_{zz} + M \in$$

$$\sigma_{yy} = Ae_{xx} + (A + 2N)e_{yy} + F\, e_{zz} + M \in$$

$$\sigma_{zz} = Fe_{xx} + F\, e_{yy} + C\, e_{zz} + Q \in$$

$$\sigma_{xy} = Ne_{xy},\ \ \sigma_{yz} = Le_{yz},\ \sigma_{zx} = Le_{zx}$$

$$S = Me_{xx} + Me_{yy} + Q\, e_{zz} + R \in \qquad (3)$$

Strain components are expressed in terms of displacements

$$e_{xx} = \frac{\partial u_x}{\partial x};\quad e_{yy} = \frac{\partial u_y}{\partial y};\ e_{zz} = \frac{\partial u_z}{\partial z}.$$

$$e_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right);$$

$$e_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right);$$

$$e_{yz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right). \qquad (4)$$

The displacements of love waves along *x*-axis and *z*-axis vanishes thus we have

$u = 0$, $w = 0$, $v = v\,(x, z, t)$ `

$U = 0$, $W = 0$, $V = V(x, z, t)$

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Combining the equations (2)-(4) with the above displacements, equation (1) reduces to

$$\left(\frac{N-P}{2}\right)\frac{\partial^2 v}{\partial x^2} + \frac{1}{2}\frac{\partial^2 v}{\partial z^2} = \frac{\partial^2}{\partial t^2}(e_{11}v + e_{12}V)$$

$$0 = \frac{\partial^2}{\partial t^2}(e_{12}v + e_{22}V) \quad (5)$$

Assuming harmonic wave solution in the form

$v_1 = f(z)\exp[i(kx + wt)]$

$V_1 = g(z)\exp[i(kx + wt)]$

where k is a wavenumber, equation (5) yields

$$\frac{L}{2}\frac{d^2 f}{dx^2} - k^2\left(\frac{N-P}{L}\right) = -w^2(\rho_{11}f + \rho_{12}g)$$

$$0 = -w^2(\rho_{12}f + \rho_{22}g) \quad (6)$$

Using the solution of the above equations, the displacement v_1 and stress σ_{yz} in the layer can be obtained as

$$v_1 = (c_1 e^{i\gamma} + c_2 e^{-i\gamma})\exp[i(kx + wt)] \quad (7)$$

where $\gamma^2 = -k^2\frac{(N-P)}{L} + \frac{2\omega^2}{L}\left(\frac{\rho_{11}\rho_{22} - \rho_{12}^2}{\rho_{22}}\right)$

2. 2 Solution in half space

Under starting stress P′, the equations of motion in an inhomogeneous elastic solid (Biot, 1965) are as follows:

$$\frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{xz}}{\partial z} - P'\left[\frac{\partial\,\omega_x'}{\partial y} - \frac{\partial\,\omega_y'}{\partial z}\right] = \rho\frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial\tau_{yx}}{\partial x} + \frac{\partial\tau_{yy}}{\partial y} + \frac{\partial\tau_{yz}}{\partial z} - P'\left[\frac{\partial\omega_z'}{\partial x}\right] = \rho\frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial\tau_{zx}}{\partial x} + \frac{\partial\tau_{zy}}{\partial y} + \frac{\partial\tau_{zy}}{\partial y} - P'\left[\frac{\partial\,\omega_y'}{\partial x}\right] = \rho\frac{\partial^2 w}{\partial t^2}$$

(8)

Where $\tau_{xx}, \tau_{xy}, \ldots..$ are incremental stresses u, v and w are displacement components

$$\omega_x' = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

$$\omega_y' = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

$$\omega_z' = \frac{1}{2}\left[\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right]$$

For love waves $u = 0, w = 0$ and $v = v(x, z, t)$ we get

$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} - \frac{P'}{2}\left[\frac{\partial^2 u}{\partial x^2}\right] = \rho\,\frac{\partial^2 u}{\partial t^2} \tag{9}$$

The non-zero stress-strain relations are

$$\tau_{yx} = \mu\, e_{xy} = \frac{\mu}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$$

$$\tau_{yx} = \mu\, e_{yz} = \frac{\mu}{2}\left[\frac{\partial w}{\partial x} + \frac{\partial v}{\partial z}\right] \tag{10}$$

In homogeneity of rigidity and density of the half-space are

$$\mu = \mu'(1 + \epsilon z), \quad \rho = \rho'(1+\xi\, z)$$

Substituting μ in equation (10),

$$\tau_{yx} = \mu' \frac{(1+\epsilon z)}{2}\frac{\partial v}{\partial x}$$

$$\tau_{yz} = \mu' \frac{(1+\epsilon z)}{2}\frac{\partial v}{\partial z}$$

Using these stresses, the equation of motion (9) reduces to

$$\left(1 - \frac{P'}{\mu'(1+\epsilon z)}\right)\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} + \left(\frac{\epsilon}{1+\epsilon z}\right)\frac{\partial v}{\partial z} = 2\,\frac{\rho'}{\mu'}\frac{(1+\xi\, z)}{(1+\epsilon z)}\frac{\partial^2 v}{\partial t^2} \tag{11}$$

Assuming wave solution in the form

$$v(z) = g(z)\, e^{i(kx+wt)}$$

and substituting it in equation (11), we get

$$\frac{d^2 g(z)}{dz^2} + \left(\frac{\epsilon}{1+\epsilon z}\right)\frac{dg}{dz} + \left(2\,\frac{\rho'}{\mu'}\frac{(1+\xi\, z)}{(1+\epsilon z)}c^2 - \left(1 - \frac{P'}{2\ \mu'(1+\epsilon z)}\right)\right)k^2 g(z) = 0 \tag{12}$$

where $c^2 = \frac{\omega^2}{k^2}$

Taking $g(z) = \frac{\emptyset(z)}{\sqrt{1+\epsilon z}}$ in above equation, we obtain

$$\frac{d^2\emptyset(z)}{dz^2} + \left(\frac{1}{4}\frac{\epsilon^2}{(1+\epsilon Z)^2} - \left(\left(1 - \frac{P'}{\mu'(1+\epsilon z)}\right) - \frac{2c^2}{c_l^2}\frac{(1+\xi\, z)}{(1+\epsilon z)}\right)k^2\right)\emptyset(z) = 0 \tag{13}$$

where $c_l^2 = \frac{\mu'}{\rho'}$

Now defining the variables $\beta^2 = 1 - \frac{P'}{\mu'(1+\epsilon z)} - \frac{2c^2}{c_l^2}\frac{\xi}{\epsilon}$ and $s = \frac{2\beta k(1+\epsilon z)}{\epsilon}$,

equation (13) can be written as

$$\frac{d^2\phi(s)}{ds^2} + \left(\frac{R}{2s} + \frac{1}{4s^2} - \frac{1}{4}\right)\phi(s) = 0 \tag{14}$$

where $R = \frac{2\omega^2(\epsilon-\xi)}{c_l^2\beta\,\epsilon^2 k} = \frac{2c^2(\epsilon-\xi)k}{c_l^2\beta\epsilon^2}$

The solution of equation (14) is

$$\phi(s) = D_1\, W_{\frac{R}{2},0}(s) + D_2\, W_{-\frac{R}{2},0}(-s) \tag{15}$$

where D_1, D_2 are constants and $W_{\frac{R}{2},0}(s)$ and $W_{-\frac{R}{2},0}(-s)$ are the Whittaker's functions. Since the solution is required in half-space, we must have $v(z) \to 0$ as $z \to \infty$ i.e. $\phi(s) \to 0$ as $s \to \infty$ and hence

$$\phi(s) = D_2\, W_{-\frac{R}{2},0}(-s)$$

Now, the displacement component $v(z)$ can be written as

$$v_2(z) = D_2\left(\frac{W_{-\frac{R}{2},0}(-s)}{(1+\epsilon Z)^{1/2}}\right) e^{i(\omega t - kx)} \tag{16}$$

Considering up to linear terms of the Whittaker's functions, equation (16) can be written as

$$v_2(z) = D_2 e^{\frac{(1+\epsilon z)\beta k}{\epsilon}}\left(\frac{2\beta}{\epsilon}\right)^{-\frac{R}{2}}(1+\epsilon z)^{-\left(\frac{R+1}{2}\right)}\left(1 - \frac{\epsilon\left(\frac{R+1}{2}\right)^2}{2\beta k(1+\epsilon z)}\right) \tag{17}$$

3. Boundary conditions and frequency equation

When there is a perfect bonding between the layer and half-space, the open boundaries are stress-free and, the displacements and normal stresses at the interface are continuous. Thus, the geometry of the problem leads to the following boundary conditions:

at $z = -h$, $\sigma_{yz} = 0$

at $z = 0$, $\sigma_{yz} = \tau_{yz}$

$$v_1(z) = v_2(z) \tag{18}$$

Equations represented in boundary conditions (18) are

$$(C_1\, i\gamma L\, e^{i\gamma z} - C_2\, i\gamma L\, e^{-i\gamma z}) = 0$$

$$(C_1\, i\gamma L - C_2\, i\gamma L - D_1\, A) = 0$$

$$(C_1 + C_2 - D_2\, B) = 0 \quad (19)$$

where $A = \frac{\mu}{2}\left(\frac{2\beta}{\epsilon}\right)^{\frac{-R}{2}} e^{\frac{-\beta k}{\epsilon}}\left[\frac{\epsilon}{2}\left(\frac{R+1}{2}\right)^2\left(1+\frac{\epsilon}{2\,\beta K}\left(\frac{R+1}{2}\right)+\frac{\epsilon}{\beta K}\right)-\beta k - \frac{\epsilon}{2}\left(\frac{R+1}{2}\right)\right]$

$$B = e^{-\frac{\beta k}{\epsilon}}\left(\frac{2\beta}{\epsilon}\right)^{\frac{-R}{2}}\left(1-\frac{\epsilon}{2\,\beta K}\left(\frac{R+1}{2}\right)^2\right) \quad (20)$$

Eliminating arbitrary constants C_1 , C_2 and D_2 in equations (18), we obtain

$$\tan\gamma h = \frac{-\frac{\mu}{2}\left(\frac{2\beta}{\epsilon}\right)^{\frac{-R}{2}} e^{\frac{-\beta k}{\epsilon}}\left[\frac{\epsilon}{2}\left(\frac{R+1}{2}\right)^2\left(1+\frac{\epsilon}{2\,\beta K}\left(\frac{R+1}{2}\right)+\frac{\epsilon}{\beta K}\right)-\beta k-\frac{\epsilon}{2}\left(\frac{R+1}{2}\right)\right]}{\gamma L\;\; e^{-\frac{\beta k}{\epsilon}}\left(\frac{2\beta}{\epsilon}\right)^{\frac{-R}{2}}\left(1-\frac{\epsilon}{2\,\beta K}\left(\frac{R+1}{2}\right)^2\right)} \quad (21)$$

where $\gamma^2 = \frac{\omega^2}{L}\left(\frac{N}{V_3^2}\right) - k^2\,\frac{(N-P)}{L}$, $\beta^2 = 1 - \frac{P'}{\mu'\,(1+\epsilon z)} - \frac{2c^2}{c_l^2}\frac{\xi}{\epsilon}$ and $s = \frac{2\beta k(1+\epsilon z)}{\epsilon}$.

Case (1) If $\epsilon \to 0$ and $\xi \to 0$ then the non-homogeneous half-space reduces to homogeneous half-space and with initially stressed transversely isotropic poroelastic layer, the secular equation (21) becomes

$$\tan\gamma h = \frac{\mu}{2L}\frac{\left[\left(1-\frac{P'}{\mu'}\right)-\frac{C^2}{C_3^2}\right]k}{\sqrt{\left(1-\frac{P'}{\mu'}\right)}\sqrt{\frac{N}{L}\frac{\omega^2}{V_3^2}-k\left(\frac{N-P}{L}\right)}} \quad (22)$$

Case (1.1) If $\epsilon \to 0$, $\xi \to 0$ and $P' \to 0$ then the half-space is homogeneous with initially stress zero and the layer is initially stressed transversely isotropic poroelastic half-space, the secular equation (21) reduces to

$$\tan\left(\sqrt{\frac{N}{L}\frac{\omega^2}{V_3^2}-\left(\frac{N-P}{L}\right)k^2}\right)h = \frac{\mu}{2L}\frac{\left[1-\frac{C^2}{C_3^2}\right]k}{\sqrt{\frac{N}{L}\frac{\omega^2}{V_3^2}-\left(\frac{N-P}{L}\right)k^2}} \quad (23)$$

Case (1.2) If $\epsilon \to 0$, $\xi \to 0$ and $P \to 0$ then the non-homogeneous half-space reduces to homogeneous half-space and transversely isotropic poroelastic layer is without initial stress. Then the modified secular equation (21) is

$$\tan\left(\sqrt{\frac{N}{L}\left(\frac{\omega^2}{V_3^2}-k^2\right)}\right)h = \frac{\mu}{2L}\frac{\left[\left(1-\frac{P'}{\mu'}\right)-\frac{C^2}{C_3^2}\right]k}{\sqrt{\left(1-\frac{P'}{\mu'}\right)}\sqrt{\sqrt{\frac{N}{L}\left(\frac{\omega^2}{V_3^2}-k^2\right)}}} \tag{24}$$

Case (1.3) If $\epsilon \to 0$, $\xi \to 0, P' \to 0$ and $P \to 0$ then the half-space reduces to homogeneous half-space without initial stress and layer is transversely isotropic poroelastic layer is without initial stress. The secular equation (21) becomes

$$\tan\left(\sqrt{\frac{N}{L}\left(\frac{\omega^2}{V_3^2}-k^2\right)}\right)h = \frac{\mu}{2L}\frac{\left[\left(1-\frac{P'}{\mu'}\right)-\frac{C^2}{C_3^2}\right]k}{\sqrt{\left(1-\frac{P'}{\mu'}\right)}\sqrt{\sqrt{\frac{N}{L}\left(\frac{\omega^2}{V_3^2}-k^2\right)}}} \tag{25}$$

Case (2)

When layer is poroelastic i.e. $L = N$ then the secular equation (21) reduces to

$$\tan\gamma h = \frac{-\frac{\mu}{2}\left(\frac{2\beta}{\epsilon}\right)^{\frac{-R}{2}} e^{\frac{-\beta k}{\epsilon}}\left[\frac{\epsilon}{2}\left(\frac{R+1}{2}\right)^2\left(1+\frac{\epsilon}{2\,\beta K}\left(\frac{R+1}{2}\right)+\frac{\epsilon}{\beta K}\right)-\beta k-\frac{\epsilon}{2}\left(\frac{R+1}{2}\right)\right]}{\gamma L\; e^{-\frac{\beta k}{\epsilon}}\left(\frac{2\beta}{\epsilon}\right)^{\frac{-R}{2}}\left(1-\frac{\epsilon}{2\,\beta K}\left(\frac{R+1}{2}\right)^2\right)} \tag{26}$$

where $\gamma^2 = \frac{\omega^2}{V_3^2} - k^2\left(1-\frac{P}{N}\right)$

and the equation (26) is the secular equation Love waves at the contact of an initially stressed poroelastic layer with a non-homogeneous half-space

Case (2.1)

If $\epsilon \to 0$ and $\xi \to 0$ then the non-homogeneous half-space reduces to homogeneous half-space in contact with poroelastic layer that is initially stressed and the secular equation (26) reduces to

$$\tan\left(\sqrt{\frac{\omega^2}{V_3^2}-k^2\left(1-\frac{P}{N}\right)}\right)h = \frac{\mu}{2L}\frac{\left[\left(1-\frac{P'}{\mu'}\right)-\frac{C^2}{C_3^2}\right]k}{\sqrt{\left(1-\frac{P'}{\mu'}\right)}\sqrt{\frac{N}{L}\frac{\omega^2}{V_3^2}-k\left(\frac{N-P}{L}\right)}} \tag{27}$$

Case (2.2) If $\epsilon \to 0$, $\xi \to 0$ and $P' \to 0$ then the half-space is homogeneous with initially stress zero and the layer is initially stressed poroelastic half-space, the secular equation (26) reduces to

$$\tan\left(\sqrt{\frac{\omega^2}{V_3^2}-k^2\left(1-\frac{P}{N}\right)}\right)h = \frac{\mu}{2L}\frac{\left[1-\frac{C^2}{C_3^2}\right]k}{\sqrt{\frac{N}{L}\frac{\omega^2}{V_3^2}-\left(\frac{N-P}{L}\right)k^2}} \tag{28}$$

Case (2.2) If $\epsilon \to 0$, $\xi \to 0$ and $P \to 0$ then the non-homogeneous half-space reduces to homogeneous half-space and isotropic poroelastic layer is without initial stress. The modified secular equation (26) is

$$\tan\left(\sqrt{\left(\frac{\omega^2}{V_3^2} - k^2\right)}\right) h = \frac{\mu}{2L} \frac{\left[\left(1 - \frac{P'}{\mu'}\right) - \frac{c^2}{c_3^2}\right] k}{\sqrt{\left(1 - \frac{P'}{\mu'}\right)} \sqrt{\sqrt{\frac{N}{L}\left(\frac{\omega^2}{V_3^2} - k^2\right)}}} \tag{29}$$

Case (2.3) If $\epsilon \to 0$, $\xi \to 0, P' \to 0$ and $P \to 0$ then the half-space is homogeneous without initial stress and layer is poroelastic layer is without initial stress. The secular equation (26) reduces to

$$\tan\left(\sqrt{\left(\frac{\omega^2}{V_3^2} - k^2\right)}\right) h = \frac{\mu}{2L} \frac{\left[1 - \frac{c^2}{c_3^2}\right] k}{\sqrt{\frac{N}{L}\frac{\omega^2}{V_3^2} - \left(\frac{N-P}{L}\right) k^2}} \tag{30}$$

4. Numerical investigation

The influence of initial stress of the transversely isotropic layer and, inhomogeneous parameters and initial stress of the in-homogeneous half-space on the propagation of Love wave is discussed. The non-dimensional phase velocity vs. the non-dimensional wave number has been calculated and presented for a number of scenarios.

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Figures 2 and 3 depict phase velocity for different values of non-dimensional in-homogeneous parameters $(ep = \varepsilon / k, si = \xi / k)$ taking constant non-dimensional initial stresses $P_1 = 0.5, P_3 = 1$ of the layer and the lower half-space respectively. In particular, Figure 2 is plotted for fixed $\xi / k = 0.1$ and different values of $\varepsilon / k = 0.3, 0.4, 0.5, 0.6$. It is observed that phase velocity decreases gradually with an increase in wave number. Phase velocity is more for higher values of the parameter ε / k. Fixed value $\varepsilon / k = 1$ and different values for $\xi / k = 0.1, 0.2, 0.3, 0.4$ are considered in figure 3. Phase velocity decreases with an increase in wave number but phase velocity is less for higher values of ξ / k. Also, it is noted that the differences in phase velocity for different values of ξ / k is little, particularly for higher values of wave number.

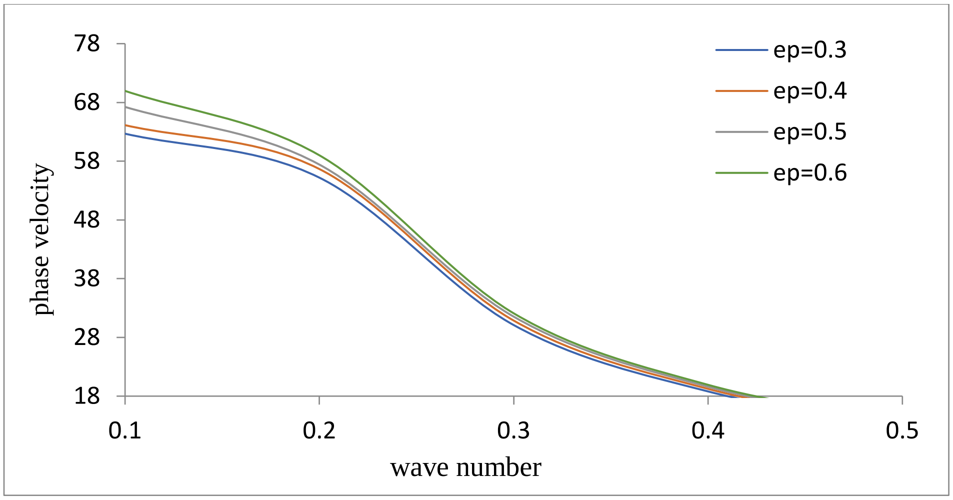

Fig. 2 Phase velocity in layer for different values of in-homogeneous parameters ε / k and constant value of $\xi / k = 0.1$.

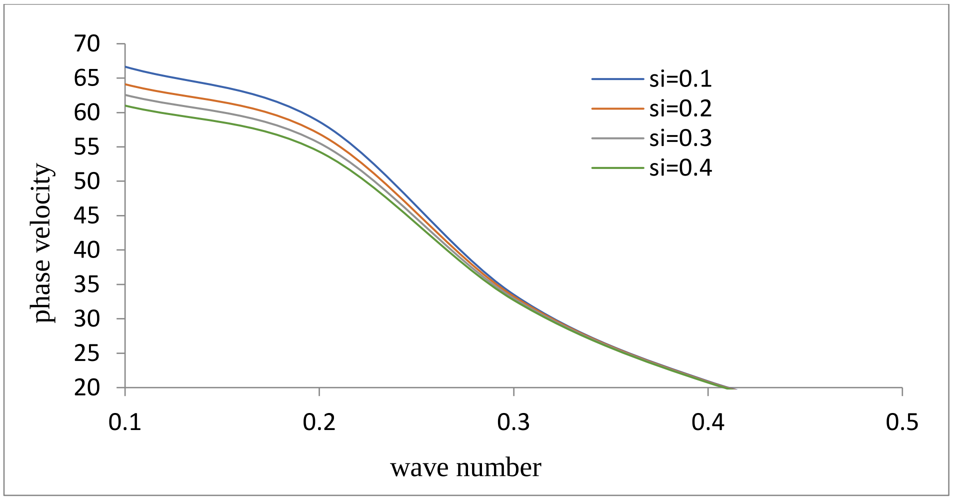

Fig. 3 Phase velocity in the layer for constant value of in-homogeneous parameters $\varepsilon / k = 1$ and different values of ξ / k.

Phase velocity in the transversely isotropic poroelastic layer against wave number for initial stress parameter $P_1 = 2$ of the layer and different values of initial stress parameter $P_2 = 0, 0.3, 0.6, 0.9, 1.2$ of lower half-space is presented in figure 4.

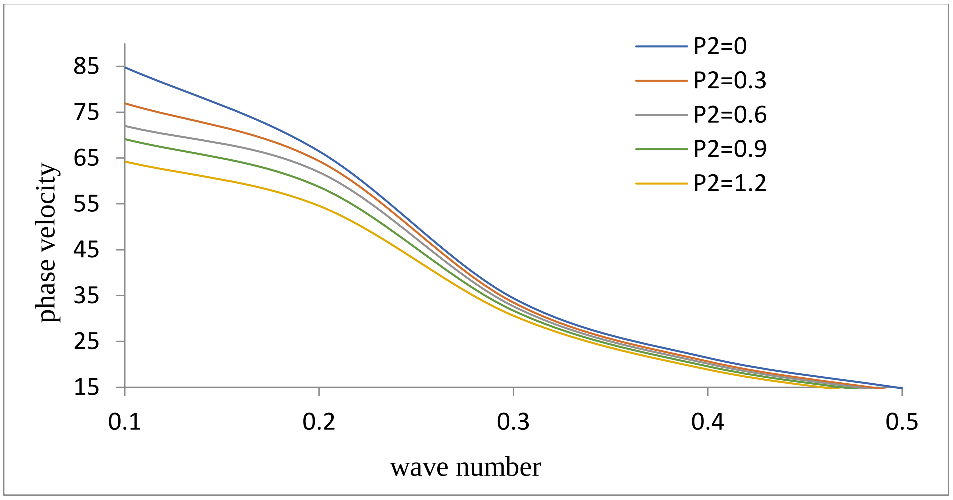

Fig. 4 Phase velocity in the layer for constant value of initial stress P_1=2 of layer and different values of initial stress P_2 of lower half-space

The non-dimensional in-homogeneous parameters are taken as $\varepsilon / k = 0.5,\ \xi / k = 0.1$. The phase velocity is low for higher values of initial stress P_2 of the lower half-space. Also, phase velocity is decreasing when wave number is increasing when initial stress P_2 of the lower half-space is taken as constant and different values of initial stress P_1 of the upper half-space is considered.

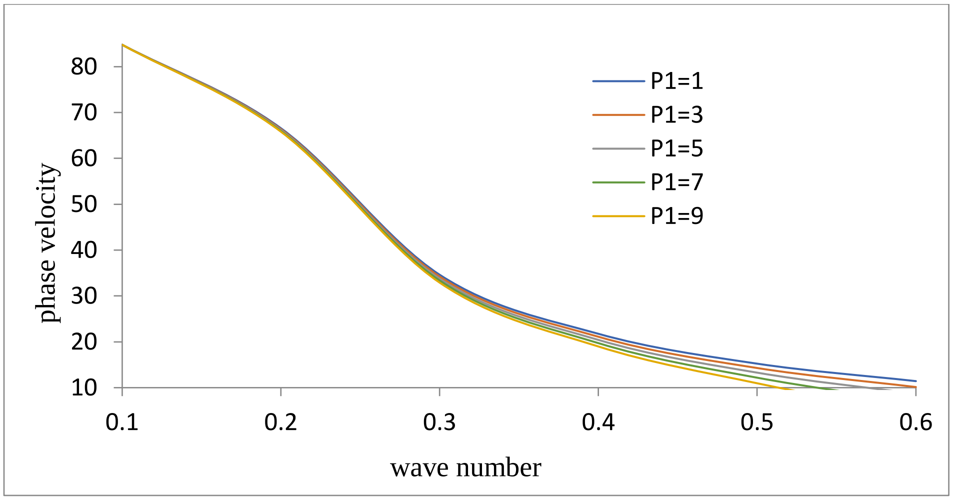

Fig. 5 Phase velocity in the layer for constant value of initial stress P_2 of the lower half-space and different values of initial stress P_1 of the layer

Figure 5 presents phase velocity for fixed initial stress parameter $P_2 = 0.1$ of the layer and different values of initial stress parameter $P_1 = 1, 3, 5, 7, 9$ of lower half-space. It is noted that the more the initial stress P_1 is taken the less phase velocity is obtained. The phase velocity is nearly same when wave number varies from 0.1 to 0.3. In figure 6, the behavior of phase velocity is presented when values of both the parameters are increasing. It is observed that phase velocity decreases as the parameter values increases. But, phase velocity decreases as the wave number increases.

Phase velocity in the layer for different combination of values of initial stresses P_1 and P_2 is presented in Figure 6. The values of P_1 are increasing whereas the values of P_2 are increasing. In particular, the values of P_1 are $9, 7, 5, 3, 1$, whereas the values of P_2 are $0, 0.3, 0.6, 0.9, 1.2$. There is dual behavior in the phase velocity for different values of wave numbers is observe.

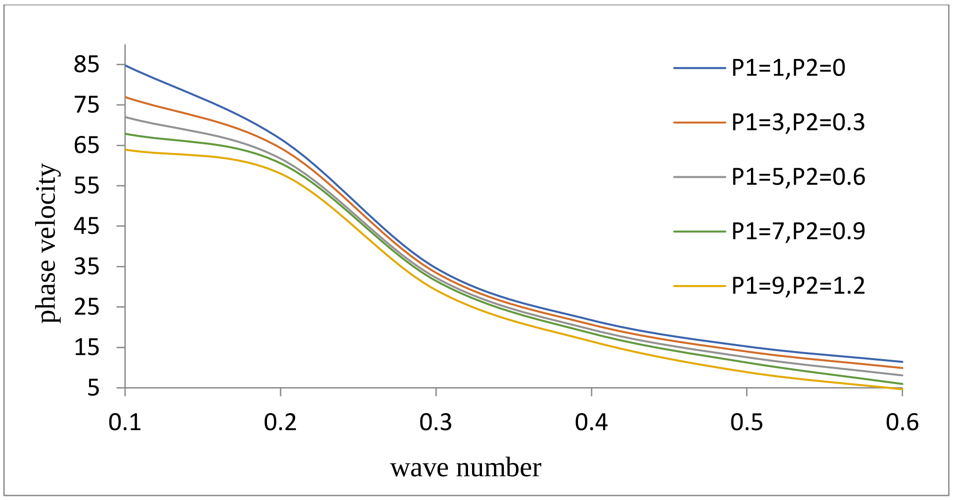

Fig. 5 Phase velocity in the layer for different values of initial stresses P_1 and P_2
The values of both parameters are increasing.

Phase velocity increases for wave numbers between 0.35 and 6, but decreases for wave numbers between 0.1 and 0.35.

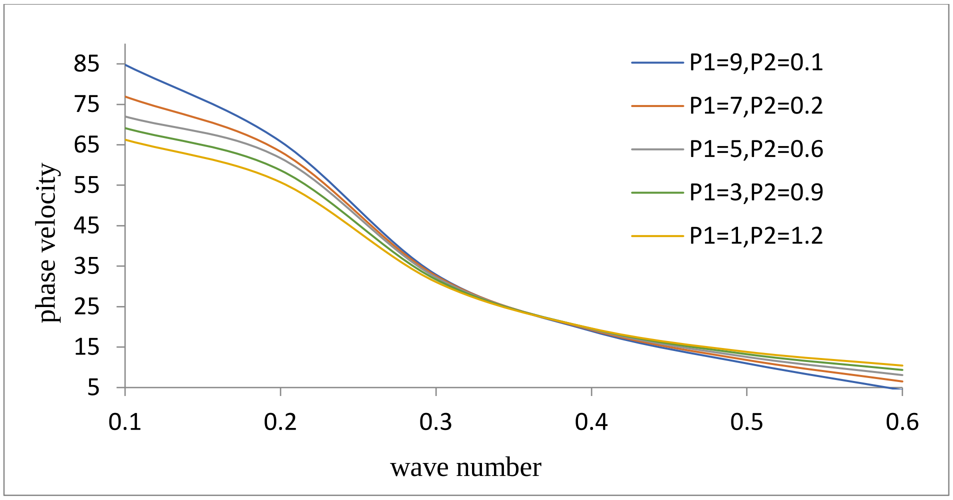

Fig. 6 Phase velocity in the layer for different values of initial stresses P_1 and P_2
The values of P_1 are increasing and the values of P_2 are increasing.

5. Conclusion

Propagation of love waves at the interface between an inhomogeneous half-space and a transversely isotropic poroelastic layer is studied. The propagation of love waves is examined in relation to initial stresses and in-homogeneity characteristics. The study leads to the following results

The phase velocity of Love waves depends on the mechanical characteristics of the materials, including their porosity, density, and shear modulus. Variations in these qualities inside the layers or at the interface result in differences in the wave dispersion characteristics. Also, the presence of initial stresses also impact the phase velocity of Love waves. Higher levels of initial stress may introduce additional stiffness or damping effects, resulting in a decrease in phase velocity, particularly at higher frequencies. Additional dispersion effects resulting from medium inhomogeneous parameters may cause the phase velocity to increase with wavenumber. Thus, Love waves usually disperse less in a homogenous medium than in an inhomogeneous one. So, the present study leads to the following results.

1. Phase velocity of Love waves decreases with an increase in wavenumber.
2. Phase velocity increases with an increase in in-homogeneous parameter ε / k
3. Phase velocity drops as inhomogeneous parameter ξ / k increases. Thus, it is evident that phase velocity increases as the ratio ξ / ε of inhomogeneous parameters increases.
4. Phase velocity is less for higher values of initial stress of lower inhomogeneous elastic half-space.
5. Phase velocity when lower half-space is homogeneous is less compared to inhomogeneous lower half-space.
6. An increase in an initial stress of lower half-space decreases the phase velocity.

7. There is no much effect of initial stress of upper half-space on phase velocity.
8. In comparison to the case of transversely isotropic upper half-space, phase velocity is higher when the upper half-space is poroelastic.

Besides offering promising prospects for applications in non-destructive testing (NDT), sensing, monitoring, and geotechnical engineering, the current study provides a rich environment for further research aimed at expanding our knowledge of wave propagation phenomena and creating novel methods and tools. But, in addition to determining the practical application of Love wave-based methodologies in real-world circumstances, experimental studies and field investigations are crucial for validating theoretical models and numerical simulations.

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